Extremal Edges: Evidence in Natural Images

Sudarshan Ramenahalli
Zanvyl Krieger Mind Brain Institute
and Dept. of Elec and Computer Eng,
Johns Hopkins University
Baltimore, MD 21218
email: sramena1@jhu.edu

Stefan Mihalas
Zanvyl Krieger Mind/Brain Institute
and Department of Neuroscience,
Johns Hopkins University
Baltimore, MD 21218
email:mihalas@jhu.edu

Ernst Niebur
Zanvyl Krieger Mind/Brain Institute
and Department of Neuroscience,
Johns Hopkins University
Baltimore, MD 21218
email:niebur@jhu.edu

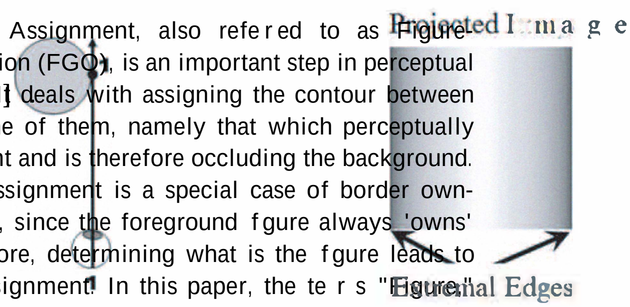
***Abstract*—In** complex natural scenes, information about figure ground organization is obtained from a combination of global ... the second PC in E2 is a T-junction in the background. The next principal component (PC2 in E1 and PC3 in E2) in both datasets has uniform intensity on the background side, and a large contrast gradient on the figure side. This is the signature of an extremal edge [1]; the gradient is a local feature indicative of an object occluding a background. To test if these components are predictive of figure-ground segregation, statistical analyses of distributions obtained by the projection of patches on the respective PCs were performed. Both datasets, E1 and E2, show significant differences on Student's t-test ($p \leq 10^{-22}$) and the Bayesian t-test ($JZSbf \leq 10^{-19}$) between figure and background distributions. We conclude that: (1) next to the T-junction the second strongest local feature in natural images predictive of figure ground organization is a luminance gradient of the figure side; (2) figure-side gradients are prevalent irrespective of the presence or absence of occlusion features.

I. INTRODUCTION

Figure-Ground Assignment, also referred to as Figure-Ground Organization (FGO), is an important step in perceptual organization [2]. It deals with assigning the contour between two regions to one of them, namely that which perceptually appears in the front and is therefore occluding the background. Figure-Ground Assignment is a special case of border ownership assignment, since the foreground figure always 'owns' the border; therefore, determining what is the figure leads to correct border assignment. In this paper, the terms "Figure", "Object" and "Foreground" are interchangeably used to refer to the region that is closer to the observer. Also "Ground" and "Background" are interchangeably used to refer to the background.

It is well established that local features such as convexity of contour, presence and location of occlusions *etc.* can, be powerful cues in determining figure-ground relationships. Many FGO cues like T-Junctions [3], symmetry [4], convexity [5], [6], familiarity [7] ... have been employed fo... [6]. In this study, we fo... to determine FGO.

Extremal edges are ... cluding object boundar... seen in Fig. 1A, an ex... of sight is tangential to... view-point specificity o... in Fig. 1 (left) and the... role of extremal-edge p... been studied [1], [9], [... organization exceeds th... convexity, shape, famili...



An important advantage of using the figure-side gradient for identifying FGO relationships is that it is a local cue and does not require access to global properties. Employing local spatial filters on small patches of the image is usually more efficient than processing global image properties, allowing for fast and efficient figure-ground assignment. Identification of extremal edges in images by a human observer is a fairly easy task. Successful application of extremal edge based cues in automatic figure-ground assignment requires that extremal edges (1) occur frequently in images, (2) are easily detectable and (3) possess unique statistical properties that can be exploited in developing figure-ground assignment algorithms. In this preliminary work, we find satisfactory evidence for claims (1), (2) and (3) through Principal Component Analysis of natural image patches.

In the next section, a brief overview of Principal Component Analysis for two-dimensional image patches is given. Section III gives an account of implementation details. We present and discuss our results in Section IV and draw conclusions in Section V.

II. 67+2#+5 $31532'2:!!2 0@9+9!

Principal Component Analysis [13], [14] reveals the underlying second-order statistical structure of high-dimensional data which might appear to lack any correlations in the original feature space. Let $\mathbf{p}_r(u,v)$ be an $N \times N$ two-dimensional gray-scale image patch and let us assume that the dataset has M patches, thus $r \in \{1, 2, \ldots, M\}$. We construct a matrix $\mathbf{X}$ which contains the vectorized $\mathbf{p}_r(u,v)$ in each column; its size is $N^2 \times M$. Its covariance matrix $\mathbf{\Sigma_X}$ is computed as:

$$\mathbf{\Sigma_X} = cov(\mathbf{X}) = (\mathbf{X} - \overline{\mathbf{X}})(\mathbf{X} - \overline{\mathbf{X}})^T / (M - 1) \tag{1}$$

which is of size $N^2 \times N^2$. Principal components and corresponding variances are obtained by eigen decomposition of $\mathbf{\Sigma_X}$ as:

$$\mathbf{\Sigma_X} = \mathrm{USU}^{-1} \tag{2}$$

where the columns of U are the principal components (eigen vectors) of $\mathbf{\Sigma_X}$ and S is a diagonal matrix whose entries are the respective variances (eigen values). By arranging the order of entries in eq. 2 such that the diagonal elements of S monotonically decrease, the columns of U are arranged to obtain the corresponding two-dimensional principal components in decreasing order. Hence, the first column in U, called the first principal component, accounts for the maximum amount of variance in the data. The second principal component is orthogonal to the first PC, it accounts for the second highest variance, and so on.

III. -150'1'2: :+32!

A total of 585 images from five different categories ['indoor'(199 images), 'office'(62 images), 'forest'(64 images), 'street'(120 images), and 'outdoor'(138 images)] were selected from the MIT LabelMe database [15]. We wanted the images to be representative of a broad range of natural scenes like beaches, indoors, offices, playgrounds, open street to avoid any systematic preselection for any specific type of patch. By doing this, we intend to reduce the effect of various biases such as illumination, frequently occurring foreground and background types, local curvatures, textures and color variations. Due to the heterogeneous nature of the database, the sizes of the images varied from 256×256 to 2048×1500 pixels. From these images, small patches of size 33×33 pixels were collected by the following procedure.

We collected patches on the boundaries between foreground and background by the process illustrated in Fig.2. On an image $I(x, y)$ the perceptual boundary of an object was visually located. On the object boundary, the center of a patch is selected and denoted as $\mathbf{p}_{center}$, it is shown as a light turquoise arrow in Fig. 2. A point tangential to the object boundary at $\mathbf{p}_{center}$ is defined and denoted as $\mathbf{p}_{tangent}$. The distinction between figure and background was also determined by visual inspection and a point $\mathbf{p}_{figure}$ is located anywhere on the figure. The green and gray arrows in Figure 2 indicate $\mathbf{p}_{tangent}$ and $\mathbf{p}_{figure}$ respectively.

(TR II \ITZDR^EHcFTSTc]b_ITRT]_DTXH\cD:cSTHf\HIXX[Eg]IhT\GTFDcHbI cSEHDI_HDD_I]e\cGSIAHDcFFSHI\cHS_DgcTIXEXH^ID_]cIcSH]cDc^HDGF:SSHI EXeDH_I_]Tg\GTFDSHBTI_HF]cTc]SHDFWR_]e\GI

The yellow box in the figure encloses an image patch in its original orientation. The blue arrow is positioned at the patch center pointing in the direction orthogonal to the object-edge. The direction of the blue arrow indicates the part of patch that forms the background.

Patches are aligned such that the part of the image on the figure side in the original image occupies the bottom half of the patch and the background the top half (*up-down orientation*). The angle θ_{rot} by which a patch is rotated in order to get it in *up-down orientation* is computed as:

$$\theta_{rot} = \angle \mathbf{v}_{FG} + \pi/2 \tag{3}$$

where the symbol $\angle \mathbf{v}$ stands for the angle between vector $\mathbf{v}$ and a fixed coordinate axis. The vector $\mathbf{v}_{FG}$ is defined as,

$$\mathbf{v}_{FG} = \mathbf{p}_{tangent} + \lambda^* (\mathbf{p}_{center} - \mathbf{p}_{tangent}) \tag{4}$$

and λ is computed as:

$$\lambda^* = \max(0, \min(\lambda, 1)) \tag{5}$$

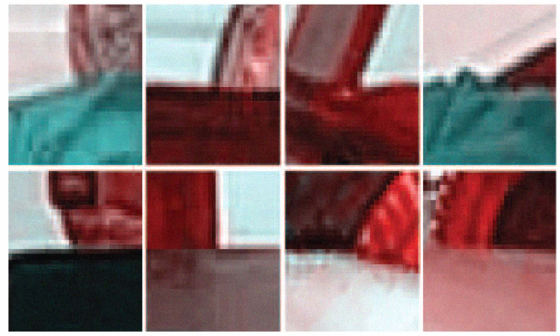
with,

$$\lambda = \frac{\langle (\mathbf{p}_{figure} - \mathbf{p}_{tangent}), (\mathbf{p}_{center} - \mathbf{p}_{tangent}) \rangle}{\langle (\mathbf{p}_{center} - \mathbf{p}_{tangent}), (\mathbf{p}_{center} - \mathbf{p}_{tangent}) \rangle} \tag{6}$$

The image patch from Fig.2 after rotation by θ_{rot} in *up-down orientation* (i.e. after rotation) is shown in Figure 3.

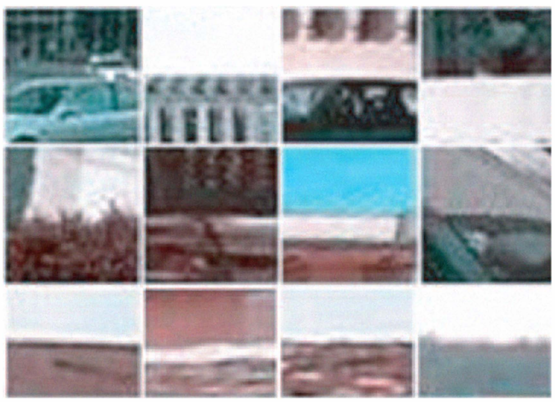
In the first experiment (EI), 'edge-only' image patches were analyzed. During their selection from the images, care was taken not to include any occlusions like T-Junctions or L-Junctions in the background. Such patches most closely resemble displays used in many psychophysical experiments [16] which typically study properties of extremal edges in the absence of occlusions in the background. We surmise that this is also the most likely scenario in natural images, in the

[illegible]



[illegible]

sense that segments usually do not include an occlusion in the background. A total of 1761 patches were used for this analysis. Some image patches from the E1 database are shown for illustrative purposes in Fig.4.



[illegible]

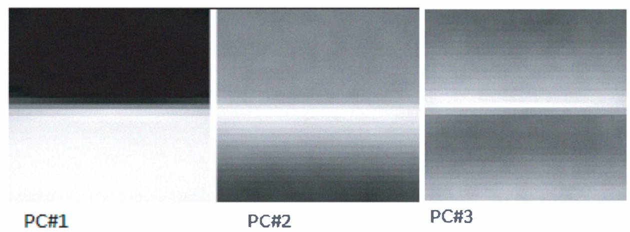
As one can notice in Fig. 4 ,some patches do not have a distinct boundary between figure and ground. This is due to the fact that in many natural scenes, a clear delineation separating figure from ground is not necessarily present in any given patch; instead, the boundary appears to be fuzzy. In the natural world such variation in edge profiles is likely to occur in many local segments, hence it was included in our dataset.

In our second experiment (E2), we study figure-ground segregration in the presence of occlusions. T-Junctions have been considered [17] to be prominent indicators of occlusion, frequently revealing the figure-ground organization of a local image patch. We study patches containing T-Junctions, some of which are shown in Figure 5. A total of 835 additional patches of T-Junctions were added to the existing dataset of E1 to obtain a combined dataset of 2596 image patches consisting of both "edge-only" patches and of patches with "T-Junctions."

The image patches were converted to 8-bit gray scale (range [0, 255]) before performing principal component analysis in both datasets, E1 and E2, separately.

IV. [illegible]

The first three principal components from the set of 'edge-only' patches (E1) are shown in Figure 6.

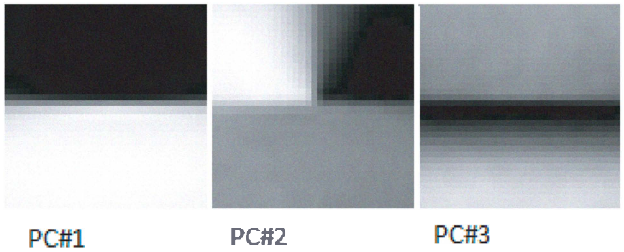


[illegible]

The first component resembles a step-edge edge, showing the contrast in luminance between figure and background. This is expected because a luminance edge is common to many patches. The second principal component is the one that is interesting to us. It has a luminance gradient on the figure-side and no such gradient in the background. This is the characteristic property of an extremal edge. PCA arranges PCs in decreasing order of variance. The fact that the signature of extremal edges is obtained already as the second principal component implies that such edges are frequently present in images when compared to other features. It is thus likely a useful cue for figure/ground assignment.

Having established the prevalence of extremal edges in natural images in the absence of occlusion, we now investigate the effect of occlusions. The first three principal components obtained in experiment (E2), in the dataset containing edges both with and without T-Junctions, are shown in Figure 7.

As in E1, the first component is a step-edge like variation in intensity, resulting from the object edge. The second component represents the T-Junction in the background, with a constant intensity on the figure side and an intensity transition orthogonal to the edge in the background. The change of intensity in the background of PC2 is not sharp, but follows a gradient.



(TR ll (T`bcS`H5#bl]`E2 D``D\RTHGHF`HD]b`G\R]llfD`TD\FHl N]ZXHO]llTRS5c# llF]``Hb^]tGDll .e\FcT]\\tSHDFWR`]5e#G l F]``Hb^]tGDb\lHhc`HZDXlHGRH l

The third PC again shows the patte r of an extremal edge, with a gradient pe r endicular to the f gure-ground boundary on the f gure side and its absence on the background side. Thus, the principal component representative of extremal edges is consistently found and it accounts for large variances in the data, both in the presence and in the absence of background occlusions.

Principal component analysis shows that correlations in pixel intensities di f er in foreground and background. The most str i ing f gure/ground di f erences are observed in the principal component corresponding to the extremal edge. Since extremal edges are present irrespective of presence or absence of occlusions (in **E 1** and in **E 2**), we are interested in studying how the principal components corresponding to extremal edges (PC2 in **E 1** and PC3 in **E 2**) correlate with image patches. We therefore determine whether there are signi f cant di f erences between the f gure side and the background side.

To this e f ect, we project image patches on to extremal edge speci f c principal components in both datasets, PC2 in **E 1** and PC3 in **E 2**. Projecting an image to a principal component is a standard way of feature extraction [18]. First, we project patches such that the f gure and background parts of the patch are in alignment with corresponding parts of the PC, we call this the ' *in-phase*' projection F_{in}. Second, the assignment is f ipped such that f gure and background sides of the PC are respectively in alignment with background and f gure sides of image patches, we call this the ' *out-of-phase*' projection F_{out}.

By carrying out such projections in both datasets **E 1** and **E 2** , four di f erent distributions, $F_{in\,E1}$, $F_{out\,E1}$, $F_{in\,E2}$ and $F_{out\,E2}$ are obtained.

We investigate whether the magnitude of distributions obtained f om *in-phase* projections of patches di f er f om those obtained f om *out-of-phase* projections. To test whether the distributions are statistically di f erent, we performed a two-tailed paired-samples Student's t-test [19]. The null hypothesis is that the two distributions are identical with the same mean. Results are summarized in Table I. The value of **h** indicates whether the null hypothesis can be rejected (for $h = 1$) or not (for $h = 0$) at our chosen signi f cance level, $\alpha = 0.05$. An estimate of the deviation of sample mean f om the hypothesized population mean ($\mu = 0$) is given by the t-statistic. Mathematically, the t-statistic is de f ned as $t = \frac{\overline{x}-\mu}{s/\sqrt{n}}$ where $\overline{x}$ is the sample mean, μ is the hypothesized mean, s is the standard deviation and n is the sample size.

	h	p	|t-statistic|	Degrees of Freedom
Edge-Only Patches(E1)	, l	2.4380×10^{-22}	l	l
Edges and T-Junctions(E2)	, l	4.9513×10^{-26}	l	l

: "0'l +l

RESULTS OF TWO-TAILED PAIRED-SAMPLES T-TESTS FOR PRINCIPAL COMPONENT RESEMBLING EXTREMAL EDGE. FOR EACH OF THE TWO EXPERIMENTS, E1 AND E2, WE CAN REJECT THE NULL HYPOTHESIS THAT THE DISTRIBUTIONS IN THE FOREGROUND AND BACKGROUND ARE THE SAME ($h = 1$).

From Table I, we can see that the distributions obtained f om *in-phase* and *out-of-phase* projections are statistically di f erent and that the di f erence is statistically (highly) signi f cant.

Student's t-test can only be used to reject the null hypothesis [20]; it can not be used to support the null hypothesis. On the other hand, a Bayesian t-test allows for accepting either the null hypothesis or the alte r ate hypothesis based on the value of the " *Bayes Factor*" [21], [22]. This factor is de f ned as the ratio of marginal densities of the data under the null and alte r ate hypotheses,

$$BF = \frac{\mathrm{P(data|H_0)}}{\mathrm{P(data|H_1)}} \tag{7}$$

If $BF < 0.01$ it strongly supports H_1, while if $BF > 100$, support for H_0 is strong (Table 1 in ref [23]). There are many ways to de f ne the Bayes Factor. We used the *JZS Bayes Factor* de f ned in ref. [20], computing it with the help of a web-based tool [24]. The results are summarized in Table II. The t-statistic computed in the paired sample t-test is used here.

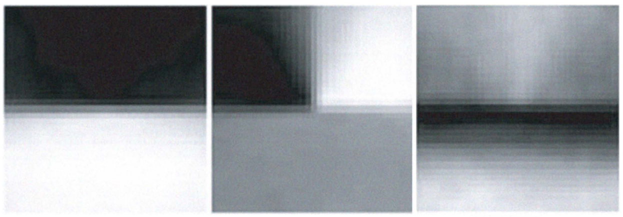
	Deg. of Freedom	|t-stat|	JZS BF	Remarks
Edge-Only Patches(E1)	l	l	l $\times 10^{-19}$	'hc`HZHfIT GH\FH`H_1
Edges and T-Junctions(E2)	l	l	l $\times 10^{-23}$	'hc`HZHfIT GH\FH`H_1

: "0'l ++l

RESULTS OF BAYESIAN T-TEST WITH A BAYES FACTOR FOR PRINCIPAL COMPONENT RESEMBLING EXTREMAL EDGE

From Table II, we can see that there is strong evidence to support the alte r ate hypothesis (that the means are not equal) as indicated by the extremely small values of the JZS Bayes Factor. Hence, we can see that *in-phase* and *out-of-phase* projections give rise to distributions that are signi f cantly di f erent.

The analyses conducted on **E 1** and **E 2** were repeated on a dataset consisting of patches in which *each* had a T-Junction in the background (835 patches total). Similar results as for **E 2** were obtained, both for the character and the order of the principal components and for the statistical tests results. The first three principal components are shown in Fig. 8. Most important is again the presence of the figure-side gradient edge in PC3. In this case, in which every single image patch has a T-junction in the background, their effect is not only seen in PC2 but a small effect carries over into PC3.



[illegible]

As in **E 1** and **E 2**, statistically significant differences between the *in-phase* and *out-of-phase* projection distributions were observed ($h = 1$; $p = 1.9354 \times 10^{-8}$; t-statistic $= 5.6729$; $JZSbj = 5.1382 \times 10^{-6}$; degrees of freedom $= 834$).

V. [illegible]

Principal component analysis reveals that extremal edges occur frequently in natural scenes. In the T-junction free dataset, the second principal component, PC2 corresponds to an extremal edge, showing that this component accounts for the second highest variance. In both data sets that allow background T-junctions, they form the second principal component. This agrees with the intuition (see also ref [3]) that T-junctions are very strong indicators of figure-ground relationships. The next principal component in this dataset, PC3, is again the one of an extremal edge. From these observations, we can conclude that (1) the prominent local feature in natural images predictive of figure ground organization that is always present along a figure-ground boundary is the pattern of an extremal edge; (2) extremal edges are prevalent irrespective of presence or absence of background T-junctions. Projections of the image patches on the principal component representing extremal edges (PC2 or PC3 without or with T-junctions, respectively) differ significantly between foreground and background. We suggest that this can be exploited in designing filters to automatically detect extremal edges, which may contribute to determine figure-ground relationships using only local information.

[illegible]

We thank Dr. Rudiger von der Heydt for many valuable suggestions and comments on the paper and Dr. Stephen E. Palmer for allowing us to use Figure 1. Work supported by Office of Naval Research grant N000141010278.

[illegible]

[illegible] *Psychological Science*, [illegible]

[illegible] *Figure and Ground: in Visual Perception*, [illegible]

[illegible]

[illegible] *Zeitschrift fur Psychologie*, [illegible]

[illegible] *Vision and Artifact*, [illegible]

[illegible]

[illegible] *Bulletin of the Psychonomic Society*, [illegible]

[illegible] *Psychological Science*, [illegible]

[illegible] *Journal of Experimental Psychology: Human Perception and Performance*, [illegible]

[illegible] *Spat. Vis.*, [illegible]

[illegible] *Journal of Vision*, [illegible]

[illegible]

[illegible] *Network: Computation in Neural Systems*, [illegible]

[illegible] *IEEE Trans. Pattern Anal. Mach. Intell.*, [illegible]

[illegible]

[illegible] *Journal of Vision*, [illegible]

[illegible] *Journal of Neurophysiology*, [illegible]

[illegible] *Journal of Cognitive Neuroscience*, [illegible]

[illegible] *Mathematical Statistics and Data Analysis.* [illegible]

[illegible] *Psychonomic Bulletin & Review*, [illegible]

[illegible] *Theory of Probability*, [illegible]

[illegible] *Journal of the American Statistical Association*, [illegible]

[illegible] *Journal of Personality and Social Psychology.*

[illegible]